The very first thing that has to be written is (depending on the size) an introduction, or an abstract, or a comprehensive title, or a statement of the objective, or ... of the report :

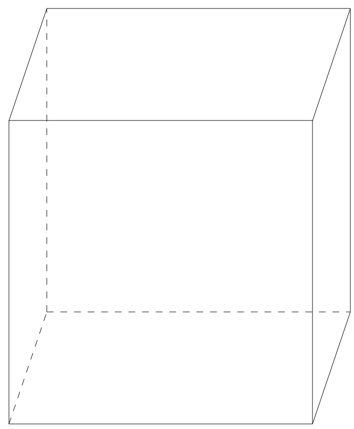
Objective : Using GeoGebra software running on a computer, draw a perspective view of a cube, containing a regular octahedron, containing a cube.

Several topics may be studied :

- *the description of the figure refering to the various definitions of the several items,*
- *the way GeoGebra is used to draw the different figures,*
- *the way GeoGebra is used to implement the rules for drawing a perspective,*
- *...*

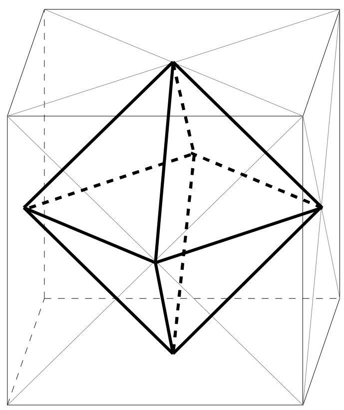
Considering the small size of the report, about one page, choices must be made, and repetition must be avoided. Sampling and producing a few examples for the selected topics might be appropriate.
Now this is about reporting on geometry work, one drawing at least is to be considered mandatory!

The hidden edges of a perspective are implemented with dotted lines : we use the « **Object properties** » window (right-click on the object), « **Style** » tab, on the line segment to be modified.



We wish to produce a clear figure, and expect a good many components, so we will choose colours from the « **Colour** » tab of the « **Object properties** » to make obvious the aim of various parts.

Every time the label of an object is unnecessary, we will hide it, for example by unchecking the « **Show label** » box in the context menu (right-click on the object).

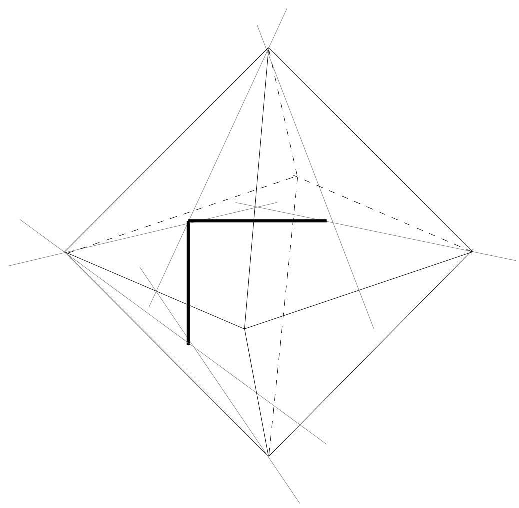


Since a cube (*synonym for regular hexahedron*) has six square faces and is regular, each centre (*or centroid*) of a face is one of the six vertices of the inscribed regular octahedron.

Plotting the centre of one face is plotting the intersection point of both its diagonals. Use GeoGebra functions : « Segment between Two Points », and « Intersect Two Objects ».

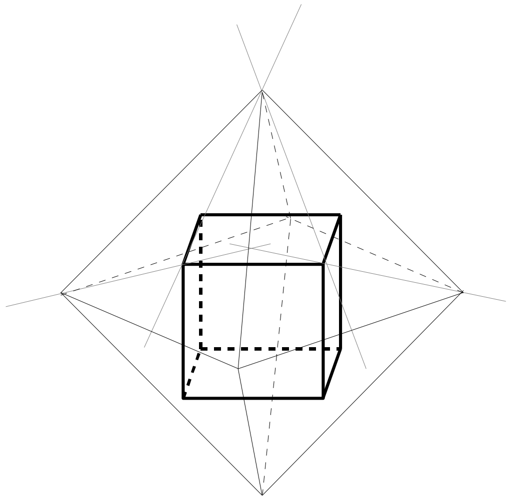
An edge of the octahedron joins a pair of vertices that belong to two adjacent faces of the cube.

Since a regular octahedron is regular and has eight triangular faces, each centroid of a face is one of the eight vertices of the inscribed cube.



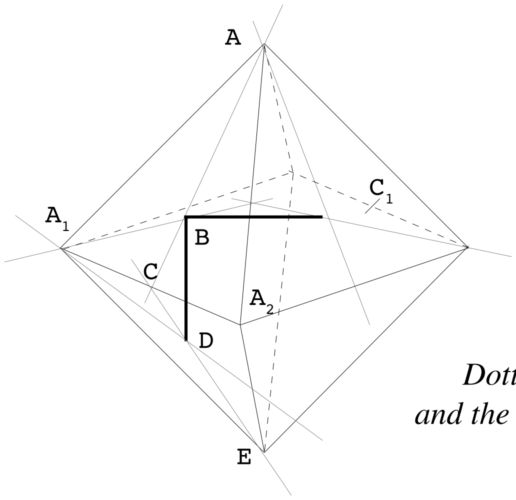
The centroid of a triangle is the intersection point of the three medians, but plotting two medians is enough. Use GeoGebra function « Midpoint or Centre » to plot the midpoint of one side, then «Line through Two Points » to plot the median through the midpoint and the opposite vertex.

An edge of the inner cube joins a pair of vertices that belong to two adjacent faces of the octahedron, unless we draw diagonals of the solid :



Proof is not asked, but several pieces may be interesting.

1 – Are the centroids of the eight faces of a regular octahedron the vertices of a cube ?

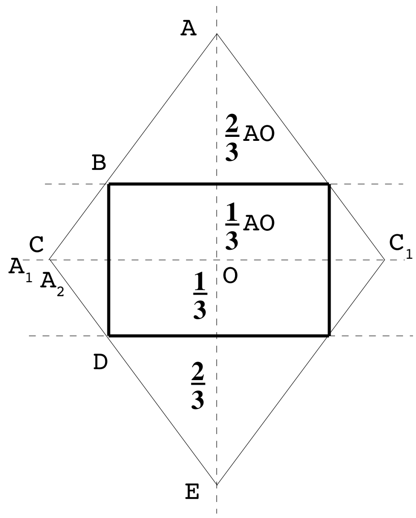


Let us draw the intersection locus of plane (ACE) *,*
polyhydron and inscribed solid :

B *is the centroid of triangular face* AA_1A_2 *, therefore :*

$$\frac{AB}{AC} = \frac{2}{3} \quad \text{and} \quad \frac{BC}{AC} = \frac{1}{3}$$

Dotted lines are parallel to (CC_1)*, and the intercept theorem provides the plotted quotients.*



[AE] *is a diagonal of the octahedron,* [BD] *is an edge of the inner solid,*

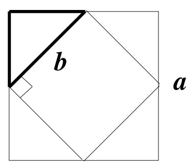
and : $BD = \frac{1}{3}AE$ *and the opposite edge of the inner solid as well.*

It is possible to cut the octahedron and the solid in five other ways so as to prove that all twelve edges of the solid are the same length $\frac{1}{3}AE$.

Parallel and perpendicular lines provide all recquired right angles to prove that the solid is a cube.

*2 - Let **a** be the length of the side of the cube. What is the length **b** of the side of the inscribed regular octahedron, in terms of **a** ?*

This is the side view of half the cube and the octahedron,
oblique segments are edges of the octahedron :



The bold isosceles right-angled triangle has sides of lenths $\frac{a}{2}$, $\frac{a}{2}$ *and **b**.*

Pythagoras' theorem states : $\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = b^2 \quad \Leftrightarrow \quad \frac{2a^2}{4} = b^2 \quad \Leftrightarrow \quad b = a\frac{\sqrt{2}}{2}$.

Since all six cuts and side views of the cube and octahedron produce the same figure, all eight edges of the octahedron are the same length: $b = a\frac{\sqrt{2}}{2}$.

*Furthermore, the inner quadrilateral has four sides of the same length **b**, its diagonals bisect each other and are the same length: it is a square. As a consequence, the angle between two faces of the octahedron is shown on the same figure, and is 90°.*

These two features, along with the congruence of the faces, are what is expected from regular polyhedra.