

### Modulus & Inequality

**Instruction :**

Transcribe the content of the tutorial video :

« Solving inequalities involving modulus : Inequalities Example 5 »

that is found at :

<http://www.livemaths.co.uk/samples/viewswf/84>

We want to find the values of  $x$  for which « the modulus of  $3x - 2$  is less than  $2x + 3$  » :

$$|3x - 2| < 2x + 3$$

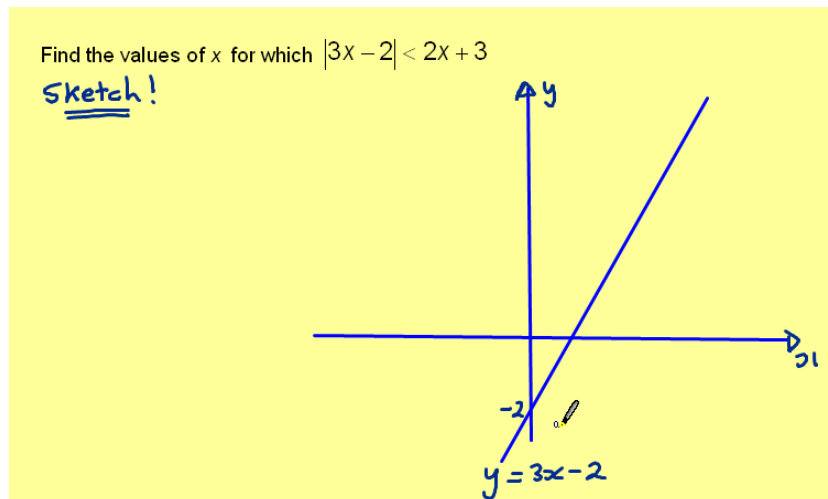
In order to make sense of this, we really need to produce a sketch.

So in order to sketch this, we need some coordinate axes. We draw our  $x$ -axis and our  $y$ -axis.

We're going to start with «  $y$  equals the modulus of  $3x$  minus  $2$  » :  $y = |3x - 2|$  .

As a first step, sketch the line «  $y$  equals  $3x$  minus  $2$  » :  $y = 3x - 2$

This is a straight line with gradient 3 which cuts the  $y$ -axis at the point  $(0 ; -2)$



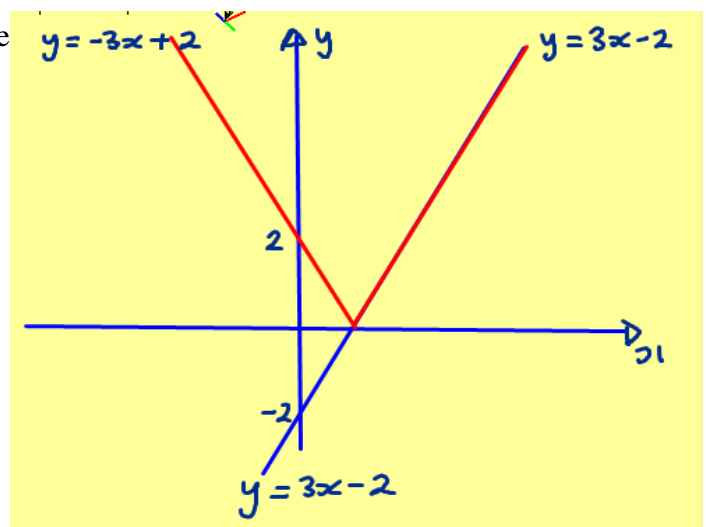
We then know, to produce the modulus, we can have no part below the  $x$ -axis.

It follows «  $y$  equals  $3x - 2$  » :  $y = 3x - 2$  down to the  $x$ -axis but then reflects upwards, and therefore cuts the  $y$ -axis at  $(0 ; 2)$

This portion of the curve is «  $y$  equals  $3x - 2$  » :

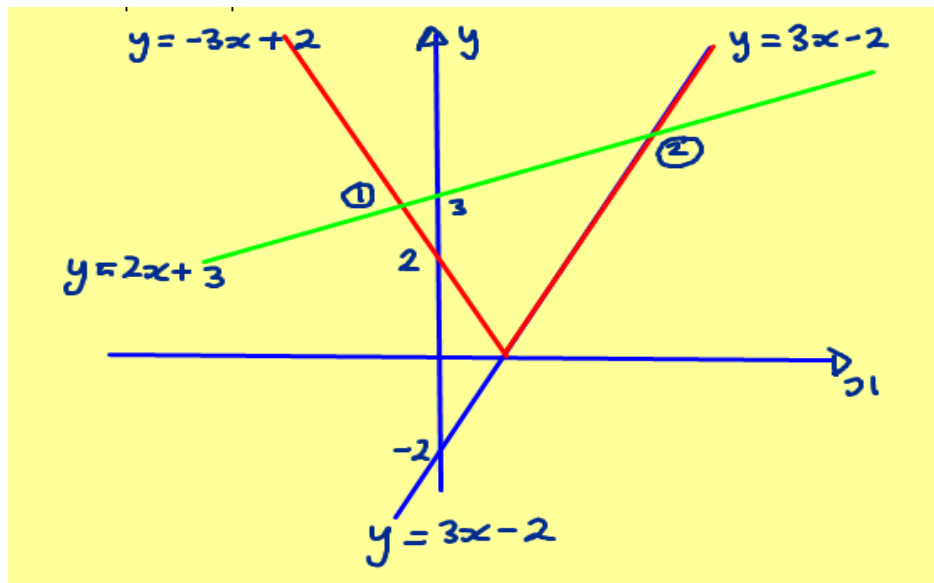
$y = 3x - 2$  and this part being a reflection in the

$x$ -axis is «  $y$  is minus  $3x$  plus  $2$  » :  $y = -3x + 2$



We now need to sketch «  $y$  equals  $2x+3$  »  $y=2x+3$ , the gradient of this will be less than the other line, and it will cut the  $y$ -axis at  $(0; 3)$

So this is «  $y$  is  $2x+3$  » :  $y=2x+3$ , we cut the axis at 3, and we can identify two points of intersection between the modulus graph and the line «  $y$  equals  $2x+3$  » :  $y=2x+3$ .



Let's consider intersection one. This is an intersection between  $-3x+2$  and  $2x+3$  so we can write down that «  $2x+3$  is equal to  $-3x+2$  » :  $2x+3 = -3x+2$

Re-arranging, we get  $5x+3=2$ , so  $5x=-1$  and «  $x$  is then equal to minus one fifth » :  $x=-1/5$

If we now consider intersection two, this is between «  $y$  is equal to  $3x-2$  » :  $y=3x-2$  and «  $y$  equals  $2x+3$  » :  $y=2x+3$

So «  $3x-2$  is equal to  $2x+3$  » :  $3x-2=2x+3$

Re-arranging, we get  $x-2=3$  and therefore  $x=5$

So the  $x$ -coordinate here we found to be  $-1/5$ .

and the  $x$ -coordinate here we found to be 5.

Since we're after the modulus being less than  $2x+3$  we're after where the modulus is below the line, in other words between  $-1/5$  and 5.

So our solution set is «  $x$  is between minus one fifth and five » :

$$-1/5 < x < 5$$

